Inverse Z-transform

$$Y(z) \xrightarrow[z^{-1}]{} y(n)$$

① Long Division (open form).

② Partial fraction (Closed form).

Ex: Given $Y(z) = \dfrac{z(z+1)}{(z-0.3)(z-0.4)(z-1)^2}$

Request: $y(0), y(1) \ldots y(4)$

Using ① Long division
② Partial fraction

Solution

$$Y(z) = \frac{z^2 + z}{z^4 - 2.7z^3 + 2.52z^2 - 0.82z + 0.12}$$

$$\begin{array}{r|l}
z^4 - 2.7z^3 + 2.52z^2 + 0.82z + 0.12 & z^2 + z \\
\hline
-(z^2 - 2.7z + 2.52 + 0.82z^{-1} + 0.12z^{-2}) & z^{-2} + 3.7z^{-3} + 7.48z^{-4} \\
\hline
3.7z - 2.52 + 0.82z^{-1} + 0.12z^{-2} & \\
-(3.7z - 10 + 9.32z^{-1} - 3.03z^{-2} + 0.44z^{-3}) & \\
\hline
7.48 - 8.5z^{-1} + 2.91z^{-2} - 0.44z^{-3} &
\end{array}$$

$$y(z) = z^{-2} + 3.7z^{-3} + 7.48z^{-4} + \ldots$$

$$Y(z) = \sum_{k=0}^{\infty} y(k) z^{-k}$$

$\Rightarrow y(0) = 0 \quad ; y(1) = 0 \quad ; y(2) = 1 \; ;$

$y(3) = 3.7 \; ; \quad y(4) = 7.48$

$$Y(z) = \frac{z(z+1)}{(z-0.4)(z-0.3)(z-1)^2}$$

obtain partial fraction

$$= Z\left[\frac{-26.53}{z-0.3} + \frac{38.88}{z-0.4} + \frac{4.76}{(z-1)^2} + \frac{-12.34}{z-1}\right]$$

$$y(n) = -26.53(0.3)^n + 38.88(0.4)^n + 4.76n - 12.34$$

$n=0 \Rightarrow \quad y(0)=0; \quad y(1)=0 \quad \ldots\ldots$

$$x_1(n) = 5\delta(n) - 2\delta(n-2)$$

$$x_2(n) = 3\delta(n-2)$$

find $y(n) = x_1(n) * x_2(n)$

① Linear Convolution

② inverse Z transform

Report